- Velocity $v(t)$: Describes the *change in position* at each time t.
- Acceleration $a(t)$: Describes the *change in velocity* at each time t.

Now that you have some calculus under your belt, that should ring some bells. We can apply our knowledge of derivatives to motion problems and solve them.

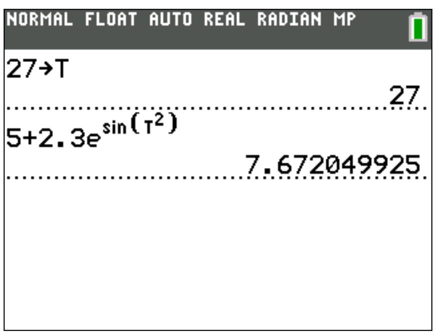
Function	Relationship
Position	$x(t)$
Velocity	$v(t) = x'(t)$
Acceleration	$a(t) = v'(t) = x''(t)$

Table 4: Relationship between functions of motion

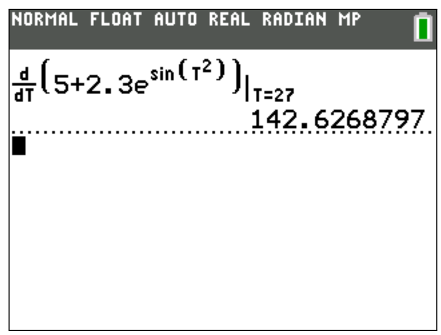
Let's apply this to an example.

For $t \geq 0$, a particle moves along a horizontal axis with a velocity that can be modeled by the function, $v(t) = 5 + 2.3e^{\sin(t^2)}$. Is the particle speeding up or slowing down at $t = 27$?

When the question asks if the particle is speeding up or slowing down, it's asking us to evaluate the acceleration. Since acceleration is the derivative of velocity, our particle will be speeding up if both acceleration and velocity have the same signs, and vice versa. Let's find $v(t)$ and $a(t)$ using our calculator. Remember that $a(t) = v'(t)$.



Finding velocity



Finding acceleration

Since both the velocity and the acceleration is positive, we can safely conclude that the object is speeding up.

3.3 Related Rates

Related rates are, first and foremost, an application of implicit differentiation. When you get crowded in context and lots of variables always keep that in mind. Let's look at several examples.

A 29 m ladder is sliding down a vertical wall so the distance between the top of the ladder and the ground is decreasing at 7 m per second. At a certain instant, the bottom of the ladder is 21 m from the wall. What is the rate of change of the distance between the bottom of the ladder and the wall at that instant?

These are the steps you *must* take to solve a related rates problem.

1. *Draw a picture.* Yes, you heard that right. Drawing a picture is extremely helpful and you should instinctively start doing it once you read the problem.

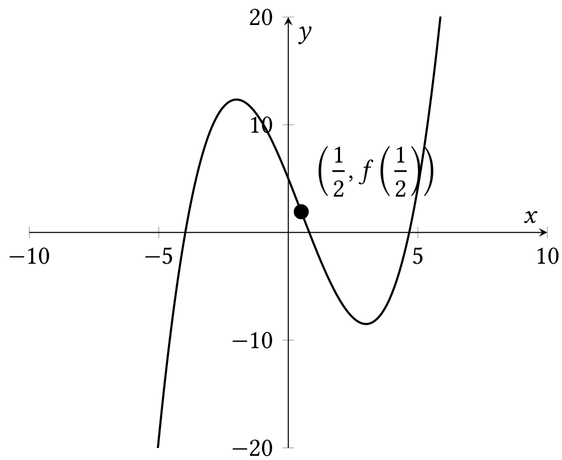
concave up and where it's concave down. First, we'll need the second derivative.

$$
\begin{aligned}
f'(x) &= x^2 - x - 6 \\
f''(x) &= 2x - 1
\end{aligned}
$$

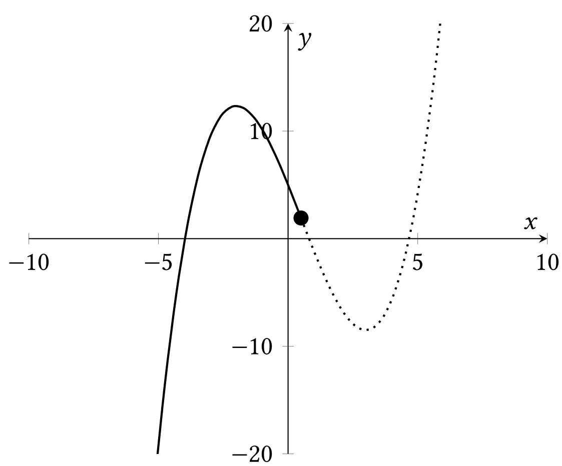
We can set the second derivative equal to 0 and solve for x.

$$
\begin{aligned}
0 &= 2x - 1 \\
x &= \frac{1}{2}
\end{aligned}
$$

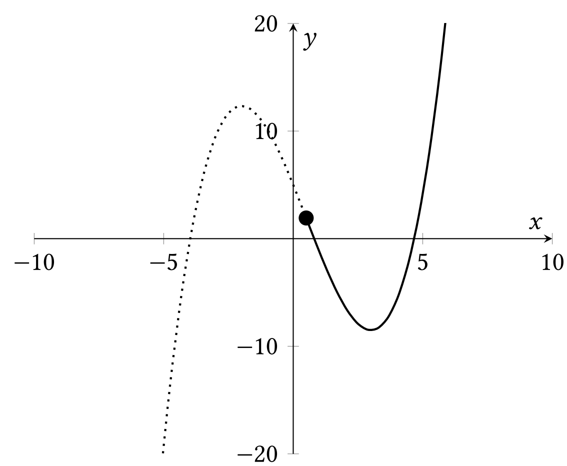
What does this point mean exactly? Let's try to visualize this.



Huh, that looks interesting. You can see that before that point, the function is concave down, but after that point, the function is concave up.

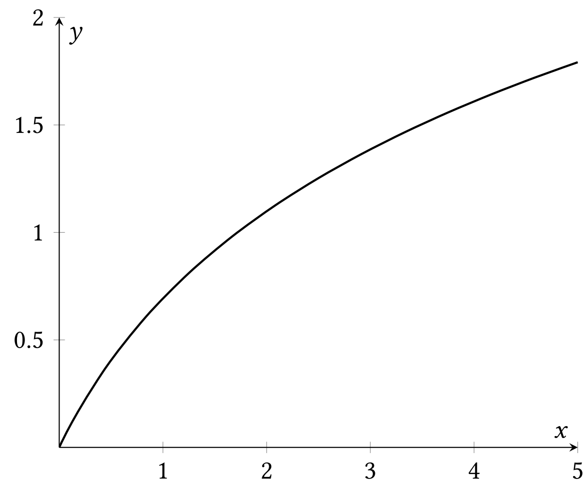


Concave down region



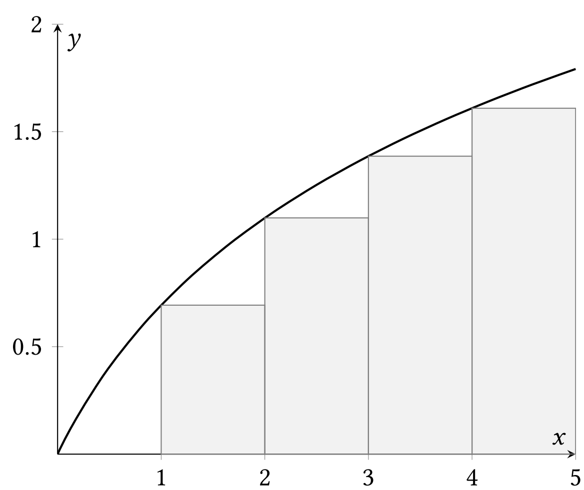
Concave up region

That point has a special name; it's called a **point of inflection**. A point of inflection is a point where the function changes its concavity. Similar to how we found the intervals of increase/decrease by plugging in x-values between the critical points, we can find the concavity on an interval by plugging in x-values between the points of inflection.

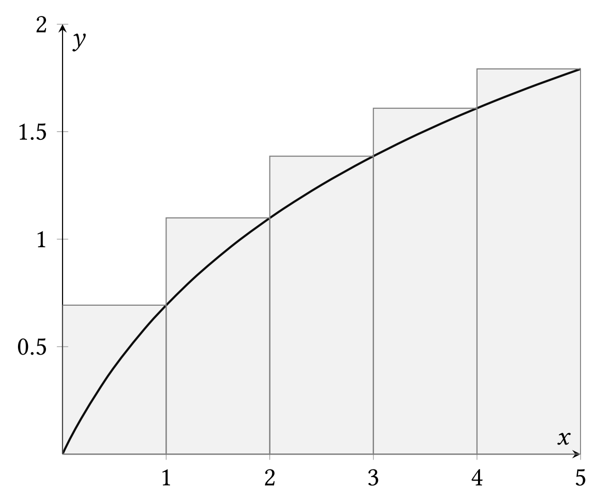


$f(x) = \ln(x + 1)$

The way that mathematicians have come to approximate this area is by splitting the area up into small rectangles and adding up their areas. These are called riemann sums.



Left-hand Riemann sum



Right-hand Riemann sum

The first one is a left-hand riemann sum because we start making rectangles from the left, and the right one is called a right-hand riemann sum because we start making rectangles from the right.

We can approximate the area under this curve using the sums above. Using the left-hand sum:

$$A \approx f(0) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 = 4.7875$$

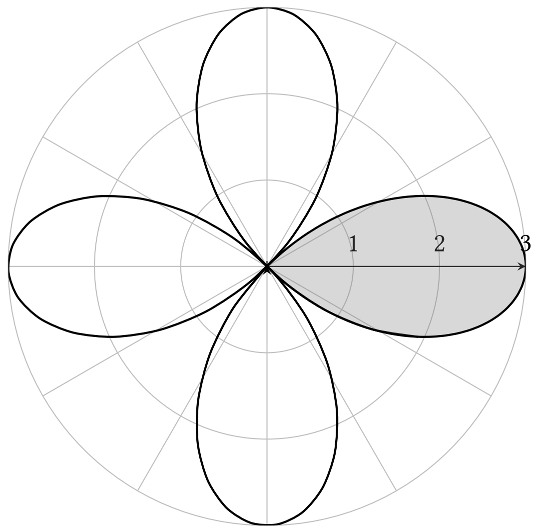
We can also try it using our right-hand riemann sum:

$$A \approx f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 + f(5) \cdot 1 = 6.5793$$

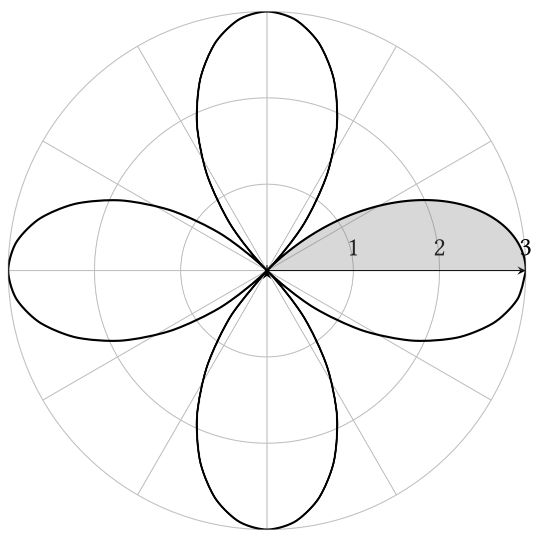
Now, the actual area under the curve is 5.7506, so you can see that both riemann sums are not quite exact, but they're in the ballpark. To increase the accuracy, we can use even more rectangles. Let's take a look at this left-hand riemann sum below, which uses 10 rectangles.

Well that's pretty straightforward. You could take the following integral, and the area it represents would resemble the following.

$$A = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (3\cos 2\theta)^2 \, d\theta$$



In fact, we can simplify it even further. We can find the area of half a petal and then double it.



The expression for that area would look like this. Since we're doubling the area of that half petal, the $\frac{1}{2}$ goes away.

$$A = \int_{0}^{\frac{\pi}{4}} (3\cos 2\theta)^2 d\theta$$